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Measurement of k_{eff} with an improved neutron source multiplication method based on numerical analysis*

YE Bin (叶滨),[1,2] ZENG Li-Na (曾丽娜),[1] YIN Yan-Peng (尹延朋),[1]
YANG Chao-Wen (杨朝文),[2] and ZHENG Chun (郑春)[1,†]

[1] *Institute of Nuclear Physics and Chemistry, China Academy of Engineering Physics, Mianyang 621900, China*
[2] *College of Physical Science and Technology, Sichuan University, Chengdu 610064, China*

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In this work, we developed a numerical analysis-associated experiment method to determine the effective multiplication factor k_{eff}, which is difficult to obtain directly from conventional neutron source multiplication (NSM) method. The method is based on the relationship between k_{eff}, subcritical multiplication factor k_{s} and external neutron source efficiency Φ^* in the subcritical system. On basis of the theoretical analysis, the dependence of k_{s} and Φ^* on subcriticality and source position was investigated at the Chinese Fast Burst Reactor-II (CFBR-II). A series of k_{s} were measured by NSM experiments at four subcritical states ($k_{\mathrm{eff}} = 0.996$, 0.994, 0.991 and 0.986) with the ^{252}Cf neutron source located at different positions (from the system center to outside) at each subcritical states. The Φ^* was obtained by Monte-Carlo simulation for each condition. With the measured k_{s} and calculated Φ^*, k_{eff} of the subcritical system was evaluated with a relative difference of $< 1\%$ between values obtained by the improved method and by positive period method. Especially, the relative difference of $< 0.18\%$ with the source located at the system center.

Keywords: k_{eff}, Neutron source multiplication, Monte-Carlo; Fast critical system

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I. INTRODUCTION

Neutron source multiplication (NSM) method, a simple measurement technique that uses an external neutron source and an ordinary neutron detector, is suitable for real-time measurement. With the recent development of accelerator-driven system (ADS) [1], an increasing attention has been paid to NSM experiments associated with various subcritical systems [2, 3]. Implementing this kind of experiments can acquire subcritical multiplication factor k_{s} and external neutron source efficiency Φ^*, which can be used to design the ADS systems, and its neutron multiplication [4, 5]. The subcritical multiplication factor k_{s} is related to the neutron multiplication M [6] by $k_{\mathrm{s}} = 1 - 1/M$, but in practice one needs to know the effective multiplication factor k_{eff}.

In this paper, in order to overcome the difficulties of getting k_{eff} directly from the conventional neutron source multiplication method, we propose a numerically associated experiment method. In Sec.II, the NSM method and the relationship between effective multiplication factor k_{eff}, subcritical multiplication factor k_{s} and neutron source efficiency Φ^* are reviewed briefly. The experiment apparatus is described in Sec. III. In Sec.IV, for verifying the proposed NSM method, the numerical analysis and a series of NSM experiments were carried out at the Chinese Fast Burst Reactor-II (CFBR-II) at various subcritical states with ^{252}Cf neutron source located at different positions in each subcritical states (from center of the system to outside).

II. THEORETICAL BASIS

For the critical systems composed of the fissile materials and other materials, the neutron balance equation in fundamental mode is expressed as:

$$\mathbf{L}\varphi_0(\mathbf{r}, E) = \frac{1}{k_{\mathrm{eff}}}\mathbf{P}\varphi_0(\mathbf{r}, E), \tag{1}$$

where, $\varphi_0(\mathbf{r}, E)$ is the neutron flux distribution at position $\mathbf{r}(x, y, z)$ at energy E, $\mathbf{L}$ is the destruction operator, and $\mathbf{P}$ is the production operator. By taking the inner product $< \ldots >$ of Eq. (1), which indicates the integration over the space and energy variables, k_{eff} in Eq. (1) can be expressed as:

$$k_{\mathrm{eff}} = \frac{< \mathbf{P}\varphi_0(\mathbf{r}, E) >}{< \mathbf{L}\varphi_0(\mathbf{r}, E) >}. \tag{2}$$

For the subcritical system with an external source at steady state, the neutron balance equation is:

$$\mathbf{L}\varphi_s(\mathbf{r}, E) = \mathbf{P}\varphi_{\mathrm{s}}(\mathbf{r}, E) + s(\mathbf{r}, E). \tag{3}$$

Similarly, the subcritical multiplication factor k_{s} can be expressed as:

$$k_{\mathrm{s}} = \frac{< \mathbf{P}\varphi_s(\mathbf{r}, E) >}{< \mathbf{L}\varphi_s(\mathbf{r}, E) >}, \tag{4}$$

where, $\varphi_{\mathrm{s}}(\mathbf{r}, E)$ is the neutron flux in the subcritical system with the external neutron source $s(\mathbf{r}, E)$.

The external source efficiency, Φ^*, is the ratio of average importance between the source neutron and fission neutron. The Φ^* implies that a neutron emitted from source, with a certain position and energy distribution, will produce certain

* Supported by Natural Science Foundation of China (No.11175164)
† Corresponding author, zhengchun@caep.cn

"important" neutrons that can lead to higher or lower neutron multiplication for a given proliferation system.

Deduced from properties of the operator and Eqs. (1)– (4), we have Eq. (5):

$$1-\frac{1}{k_{\text{eff}}}=\Phi^*\left(1-\frac{1}{k_{\text{s}}}\right)\Rightarrow k_{\text{eff}}=\frac{k_{\text{s}}}{k_{\text{s}}-\Phi^*(k_{\text{s}}-1)}. \quad (5)$$

Then, the multiplication factors k_{s} and k_{eff} can be theoretically correlated with the neutron source efficiency Φ^*.

Obtaining k_{eff} of the system through Eq. (5) needs to know the k_{s} and Φ^*. As mentioned above, the k_{s} can be measured by neutron multiplication experiment, but the quantity of neutron source efficiency Φ^* cannot be obtained from experiments directly. In the following sections, we acquired Φ^* from numerical analysis and evaluated k_{eff} successfully.

III. EXPERIMENT APPARATUS REVIEW

Experiments were carried out at the Chinese Fast Burst Reactor-II (CFBR-II) [7], an ellipsoidal assembly, which consists of, from inside to outside, the enriched-uranium spherical shells, inner brass reflector, depleted uranium reflector and outer brass reflector. The reactor can be divided into the upper, middle and lower parts (Fig. 1). The middle part is a 5.2 cm thick stainless steel plate, which has three horizontal holes for three control rods to pass through, i.e. an auto- adjustment rod, a composition rod and a pulse rod. The three rods are made of casted enriched-uranium. A 10 cm × 4.2 cm × 14.5 cm ($w \times h \times d$) irradiation chamber and two horizontal experimental holes are located in the plate. The neutron source can be put at any position in the horizontal holes by a neutron source transmission device. Reactivity of the assembly can be controlled by adjusting insert depth of the three control rods. The assembly works at subcritical, delayed critical or super prompt critical states.

A ^{252}Cf spontaneous fission neutron source (in activity of $1.41\times10^{-5}\,\text{s}^{-1}$) sealed in an aluminum box of Φ7.8 mm × 11 mm can be sent into a horizontal hole of the stainless steel plate by the transmission device when the system is in deep subcritical state. Then, the control rods are inserted to different depths to operate the reactor at different subcritical power states.

Two BF_3 proportional counters, of the SZJ-1 type, produced by Beijing Nuclear Instrument Factory, are used to record the neutrons (which is proportional to the neutron density in the system). They consist of the paraffin barrel (Hanson long counter), preamplifier, HV power supply, main amplifier and multi-scalar. The detectors are 180 cm away from the system center.

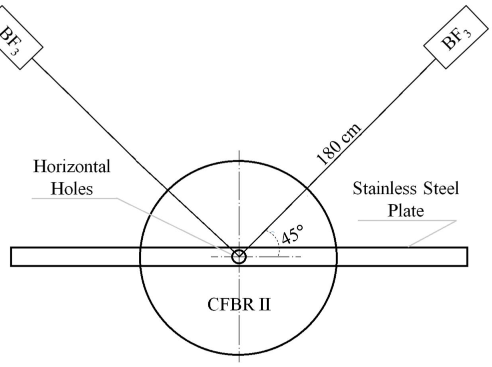

Fig. 1. Schematic of CFBR-II.

IV. NUMERICAL ANALYSIS AND EXPERIMENTS

A. Numerical calculations

Before a series of experiments are carried out, Monte-Carlo numerical method [8] was used to build the subcritical system model that is identical to CFBR-II in Fig. 1. Four subcritical states of k_{eff} =0.996, 0.994, 0.991 and 0.986 were selected, which are conditions for our multiplication experiments. The source positions were changed along the r-axis at each subcritical state (i.e. 0 mm, 20 mm, 40 mm, 60 mm, 80 mm, 100 mm, 120 mm and 150 mm). The spontaneous ^{252}Cf neutron source, described by Watt fission spectrum [9], is positioned the same way as that in the planned experiments. The k_{s} of all the experimental conditions were calculated by MCNP5 with statistic difference of $< 0.7\%$. The neutron source efficiency Φ^* is closely connected with neutron importance, which is proportional to the detector response to a unit source. In other words, neutron importance is a measure of the "importance" of a neutron in contributing to the detector response, or the expected counts per neutron in certain position, direction and energy. The detector counting rates to the system with the ^{252}Cf neutron source in the core could be obtained, and the counting rate to the eigen neutron source distributed in the system could be recorded, too, by using MCNP code. The ratio of the two counting rates should be the external neutron source-related average efficiency Φ^* [10, 11]. In terms of eigen-distribution neutron source acquisition, we used the KCODE and SSW cards in MCNP5 critical calculation to achieve the goal. Thus, we tallied Φ^* in MCNP5 calculation following certain regulation according to MCNP rules.

By calculating the k_{s}, Φ^* and k_{eff} inside and outside of the neutron source, we were able to investigate the effect of system subcriticality and source position.

For the subcritical states ($k_{\text{eff}} = 0.994$), the effect of the source positions along the r-axis are demonstrated by the calculation results in Fig. 2. The source efficiency Φ^* and subcritical multiplication factor k_{s} at different source positions indicate that the smallest discrepancy between k_{s} and k_{eff} is achieved when the source is at the center of the core.

Table 1 shows the subcriticality effect on k_{s} and Φ^* at k_{eff} =0.996, 0.994, 0.991 and 0.986. One sees that for the four subcritical states, when the source is at the core center, k_{s} is very close to k_{eff}, with a relative difference of $< 0.8\%$, and the Φ^* was greater than 0.91. The results indicate that the closer the system reaches to the critical state, the more likeliness the relative values of φ_{s} and φ_0 become equal to each

other, and so do the values of k_s and $k_{\rm eff}$. Inversely, placing the source outside the core, the discrepancy between k_s and $k_{\rm eff}$ becomes much larger, with the maximum relative difference being 22.1340% at $k_{\rm eff} = 0.986$ and the source being 150 mm away from the core center.

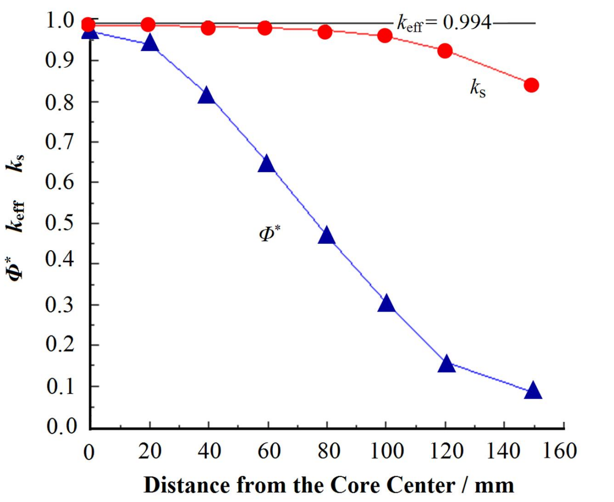

Fig. 2. (Color online) Effect of source position on subcritical multiplication factor k_s and source efficiency Φ^* at a fixed subcritical state of $k_{\rm eff} = 0.994$.

TABLE 1. Summary of subcriticality effect on k_s and Φ^* for a ^{252}Cf neutron source in the subcritical model

case	$k_{\rm eff}$	Source position (mm)	$(k_s/k_{\rm eff} - 1)$ (%)	Φ^*
1	0.9960	0	−0.6436	0.9599
		150	−12.2234	0.0251
2	0.9940	0	−0.5851	0.9272
		150	−15.2628	0.0887
3	0.9910	0	−0.7513	0.9471
		150	−18.6502	0.0939
4	0.9860	0	−0.7852	0.9176
		150	−22.1340	0.1036

In order to check the source position effects on the neutron leakage rate, we did another numerical analysis to investigate the detector response to non-collided neutrons emitted from the source, and to total neutrons. The results in Fig. 3 show that the relative difference is below 0.25%. Placing the neutron source closer to the system edge, the counting rate for non-collided neutrons from the source becomes greater, while the counts of total neutrons decrease. In other words, the detector counting rate decreases for the eigen-distribution neutron (fission neutron inside the system). This explains why the discrepancy between k_s and $k_{\rm eff}$ becomes large while moving the ^{252}Cf neutron source away from the system center.

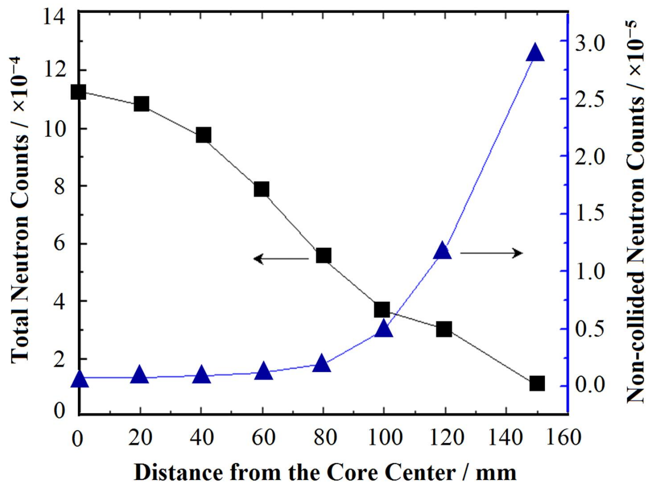

Fig. 3. (Color online) The detector counts for total and non-collided neutrons ($k_{\rm eff} = 0.994$).

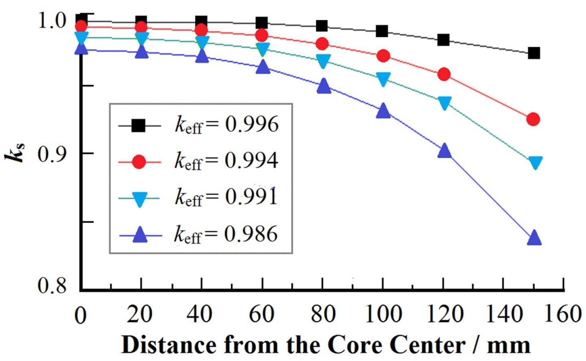

Fig. 4. (Color online) k_s measured at different subcritical states.

B. Experiments Results and Discussion

On the basis of the numerical investigation in Sec.IV.A, the NSM experiments were carried out at CFBR-II to measure k_s and evaluate Φ^*. The subcritical states were experimentally achieved by adjusting insert depth of the control rods. All the experiment apparatus and conditions were completely identical to the numerical ones. The results are shown in Fig. 4. The k_s decreased with increasing distance of the source from the core center and with deepening subcriticality. This is identical to the calculation results in Fig. 2.

In order to test the feasibility of this improved NSM method, we considered four subcritical states ($k_{\rm eff} = 0.996$, 0.994, 0.991 and 0.986) and eight neutron source positions according to each subcritical states. For the sake of the calculated source efficiency Φ^* and the measured k_s, we obtained the effective multiplication factor $k_{\rm eff}$ according to Eq. (5) at 32 studied conditions. For comparison, we used the positive periodic method to measure the reactivity as the reference method in the work. The specific process is that we measured reactivity in super-criticality, and extrapolated to subcriticality with calibrated reactivity curve of control rods.

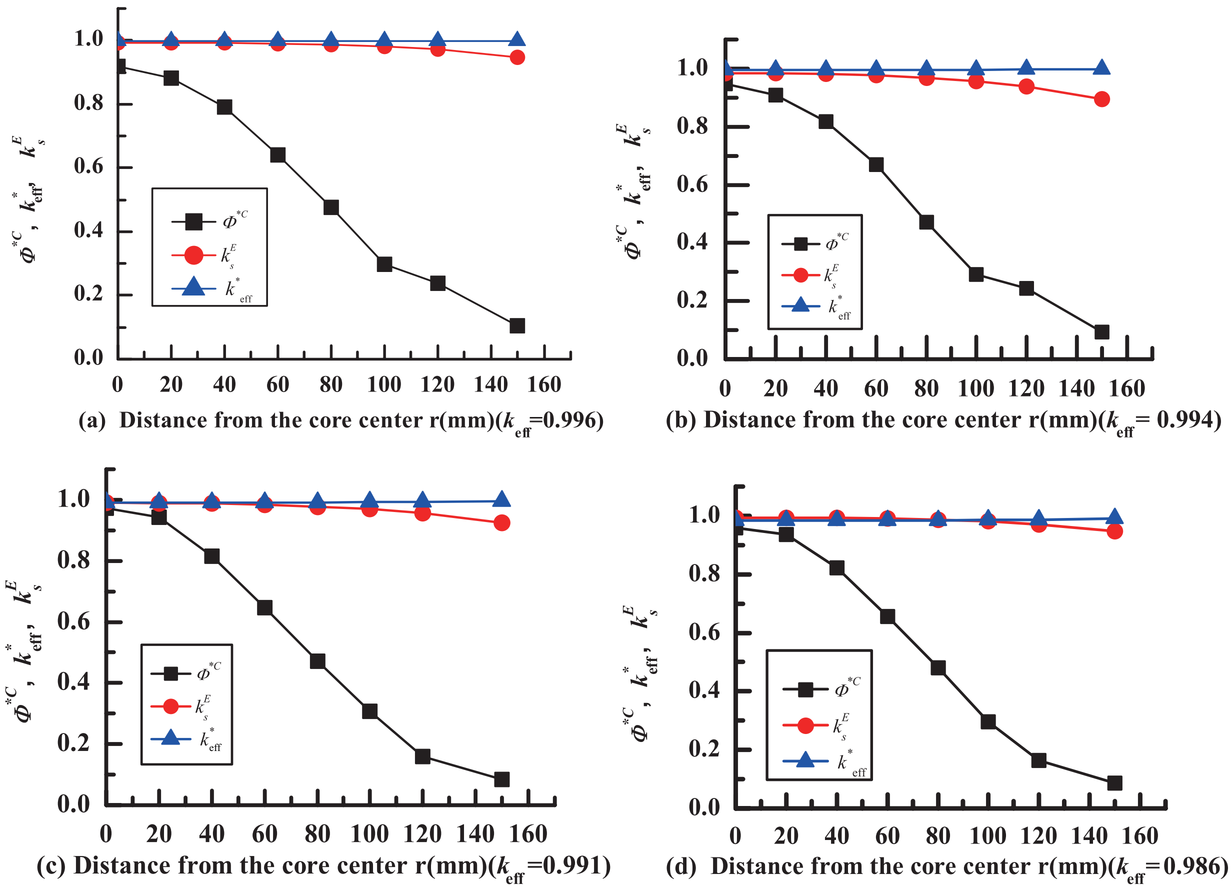

Fig. 5. (Color online) Experiments and calculations results (superscript "C": Calculated value; superscript "E": Experiments value; superscript "*": Evaluated value).

As shown in Table 2, a comparison between the reference method and the results in this work at the four subcritical conditions with the source located at the core center revealed fairly good agreement with a maximum relative difference of 0.18%. For a certain subcriticality ($k_{\rm eff} = 0.991$), the discrepancy between $k_{\rm eff}$ measured by positive period method and the evaluated ones in this work is the smallest of all. When $k_{\rm eff} > 0.9910$, the evaluated $k_{\rm eff}$ are a bit larger than experimental measured ones. However, at a very deep subcriticality, e.g. $k_{\rm eff} = 0.9860$, the evaluated $k_{\rm eff}$ is a little less than the measured one. This is because the source neutrons could not propagate efficiently in deep subcriticality. The results demonstrated the good precision of this improved method and the validation of the Monte-Carlo calculation model.

In general, $k_{\rm eff}$ of the system for all studied cases were well approximated as shown in Fig. 5.

The comparison between the $k_{\rm eff}$ results in references (positive period method, marked as $k_{\rm eff-exp}$) and our method ($k_{\rm eff-eva}$) is done by $\Delta\epsilon = [(k_{\rm eff-eva} - k_{\rm eff-exp})/k_{\rm eff-exp}] \times 100\%$. As shown in Fig. 6, the space effect on $k_{\rm eff}$ can be seen in a large range of the neutron position. Generally, when the neutron source is inside the subcritical system, the relative difference is small, comparing to that when the source is outside the system; and the relative difference increase rapidly with distance of the neutron source to the system center, being irregularly large at the distance of > 85 cm, because this is where the brass and depleted uranium reflectors locate, causing uncertainty factor related to complicated neutron behavior.

TABLE 2. Comparison between measured and calculated $k_{\rm eff}$, $k_{\rm s}$ and Φ^* in the fast critical assembly experiments while neutron source put at the center of core.

Case	$k_{\rm eff}$ [a]	$k_{\rm eff}$ [b]	$k_{\rm s}$ [c]	Φ^* [d]	$(k_{\rm eff}^{\rm a}/k_{\rm eff}^{\rm b} - 1)$ / %
1	0.9960	0.9977	0.9936	0.9599	0.1707
2	0.9940	0.9948	0.9899	0.9272	0.0805
3	0.9910	0.9915	0.9847	0.9471	0.0505
4	0.9860	0.9844	0.9757	0.9176	−0.1623

[a] Positive periodic method.
[b] Calculated in this work.
[c] Neutron Source Multiplication experiments and Eq. (1).
[d] Numerical analyzed method described in Sec.IV.A.

Therefore, to reduce the relative difference in experiments implemented, the neutron source should be put inside the subcritical system.

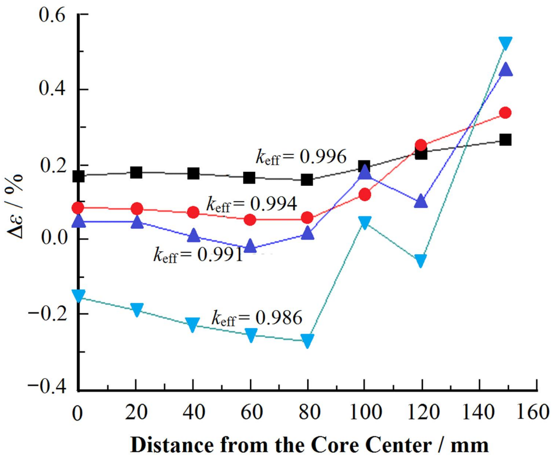

Fig. 6. (Color online) Relative difference at four subcritical states between calculated and measured ones.

V. CONCLUSION

A methodology to evaluate the effective multiplication factor k_{eff} from measured k_s and numerical analyzed Φ^* was investigated via NSM experiments on CFBR-II. The evaluated results were approximately equal to the reference method (positive period method).

On basis of the theoretical and numerical preparations, k_s was measured through a series of NSM experiments with a ^{252}Cf spontaneous fission neutron source located at different positions in four subcritical states of CFBR-II. The effective neutron multiplication factor k_{eff} obtained in all the subcritical states when neutron source located inside of the subcritical system, within a relative difference of $< 0.2\%$ between the values obtained by the improved NSM method and positive period method. So, to reduce the relative difference in experiments implemented, the neutron source should be put inside the system. The numerical analysis-associated NSM we proposed is feasible for the simple subcritical apparatus, such as CFBR-II.

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